ON FUZZY e-CONTINUOUS MULTIFUNCTIONS

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Abstract : In this paper a new type of fuzzy multifunction termed as fuzzy e-continuous multifunction has been introduced and studied. Some characterizations and several properties of fuzzy lower and upper e-continuous multifunctions are obtained.

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1.Introduction

The concept of fuzzy set was introduced by Zadeh [6]. Based on the concept of fuzzy sets, Chang [2] introduced and developed the concept of fuzzy topological spaces. Since then various important notion in the classical topology such as continuous functions [2] have been extended to fuzzy topological spaces. Fuzzy continuity is one of the main topics in fuzzy topology. Various authors introduced various types of fuzzy continuity. One of them is fuzzy e-continuity. In 2014, Seenivasan [5] introduced the concept of fuzzy e-open and fuzzy e-continuity. Throughout this paper spaces (X, δ) and (Y, σ) (or simply X and Y) represent nonempty fuzzy topological spaces due to change [2] and the symbols I and I^Xhave been used for the unit closed interval [0, 1] and the set of all functions with domain X and codomain I, respectively. The support of a fuzzy set A is the set $\{x \in X : A(x) > 0\}$and is denoted by *supp*(A).A fuzzy set with only nonzero value $p \in (0, 1]$ at only one element $x \in X$ is called a fuzzy point and is denoted by $x_p$and the set of all fuzzy points of a fuzzy topological space is denoted by $Pt(X)$. For any two fuzzy sets A and B of X,$A \leq B$ if and only if $A(x) \leq B(x)$ for all $x \in X$. A fuzzy point x_p is said to be in a fuzzy set A (denoted by$x_p \in A$) if$x_p \leq A$, that is, if $p \leq A(x)$.The set of all fuzzy points having nonzero value $\epsilon, 0 < \epsilon \leq 1$ and contained in the fuzzy set A is denoted by $\Pr(A, \epsilon)$. The constant fuzzy sets of X with values 0 and 1 are denoted by $\overline{0}$ and $\overline{1}$, respectively. A fuzzy set A is said to be quasi-coincident with B (written as $A\overline{q}B$) [3] if $A(x) + B(x) > 1$ for some$x \in X$. A fuzzy set A is said to be not quasi-coincident with B (written as$A\bar{q}B$) [3] if $A(x) + B(x) \leq 1$ for all $x \in X$. A fuzzy open set A of X is called fuzzy quasi neighborhood of a fuzzy point x_p if $x_p\bar{\mathbf{q}}A$ and the collection of all fuzzy quasi neighborhood of a fuzzy point x_p is denoted by FQN(X, x_p).The fuzzy closure of A, fuzzy interior of A, fuzzy δ-closure of A and the fuzzy δ-interior of A are denoted by $Cl(A)$, $Int(A)$, $Cl_\delta(A)$ and Int_δ(A) respectively.

A fuzzy subset A of space $X$is called fuzzy regular open [1] (resp. fuzzy regular closed) if $A = Int(Cl(A))$ (resp.$A = Cl(Int(A))$. The fuzzy δ-interior of fuzzy subset A of X is the union of all fuzzy regular open sets contained in A. A fuzzy subset A is called fuzzy δ-open [4] if A = Int_δ(A). The complement of fuzzy δ-open set is called fuzzy δ-closed (i.e, A = Cl_δ(A)). In this paper we use fuzzy e-open sets in order to obtain certain characterizations and properties of upper (lower) fuzzy e-continuous multifunction.

2. Preliminaries

Definition 2.1. [5] A fuzzy set λ of a fuzzy topological space X is said to be fuzzy e-open if $\lambda \leq Cl(Int_\delta\lambda) \vee intCl_\delta(\lambda)$, where $Cl(\lambda) = \wedge\ \{\mu: \mu \geq \lambda,$ if μ is fuzzy closed in $X\}$ and $Int(\lambda) = \vee\ \{\mu : \mu \leq \lambda,\ \mu$ is fuzzy open in $X\}$. If λ is fuzzy e-open, then $1 - \lambda$ is fuzzy e-closed.

Definition 2.2. [5] Let Xbe a fuzzy topological space and λ be any fuzzy set in X. The fuzzy e-closure of λ in X is denoted by $eCl(\lambda)$ as follows: $eCl(\mu) = \wedge\ \{\lambda: \lambda \geq \mu, \lambda$ is a fuzzy e-closed set of $X\}$. Similarly we can define $eInt(\lambda)$.

Definition 2.3. [1] A fuzzy topological space $X$is product related to a fuzzy topological space Y if for fuzzy sets δ of X and η of Y whenever $1 - \lambda \not\geq \delta$and $1 - \mu \not\geq \eta \Rightarrow (1 - \lambda \times 1) \wedge (1 \times 1 - \mu) \geq \delta \times \eta$
, where λ is a fuzzy openset in X andμ is a fuzzy open set in Y, there exists λ_1 a fuzzy open set in$X$ and μ_1 a fuzzy open set in Y such that $1 - \lambda_1 \geq \delta$ and $1 - \mu_1 \geq \eta$and
$(1 - \lambda_1 \times 1) \vee (1 \times 1 - \mu_1) = (1 - \lambda \times 1) \vee (1 \times 1 - \mu)$.

3. Fuzzy e-continuous multifunctions

Definition 3.1. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, ν). Then it is said that F is:

(1) Upper fuzzy e-continuous at $x_\epsilon \in X$if for each fuzzy open set μ of Y containing $F(x_\epsilon)$, there exists a fuzzy e-open set ρ containing x_ϵ suchthat $\rho \leq F^+(\mu)$.
(2) Lower fuzzy e-continuous at x_ϵ∈ X if for each fuzzy open set μ of Ysuch that $x_\epsilon \in F^-(\mu)$ there exists a fuzzy e-open set ρ containing$x_\epsilon$such that $\rho \leq F^-(\mu)$.
(3) Upper (lower) fuzzy e-continuous if it has this property at each pointof X.

We know that a net $(x^e_{\epsilon_e})$ in a fuzzy topological space (X, τ) is said to be eventually in the fuzzy set $\rho \leq X$ if there exists an index $e_0 \in J$ such that$(x^e_{\epsilon_e}) \in \rho$ for all $e \geq e_0$.

Definition 3.2.A sequence (x_{ϵ_n}) is said to e-converge to a point X if for everyfuzzy e-open set μ-containing $x_\epsilon$there exists an index n_0 such that for

$n \geq n_0, (x_{\epsilon_n}) \in \mu$.This is denoted by $(x_{\epsilon_n}) \to_e x_\epsilon$.

Theorem 3.1. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, ν).Then the following statements areequivalent:

(1) F is upper fuzzy e-continuous.
(2) For each $x_\epsilon \in X$ and for each fuzzy open setμ such that $x_\epsilon \in F^+(\mu)$ there exists a fuzzy e-open set ρ containing x_ϵ such that $\rho \leq F^+(\mu)$.
(3) $F^+(\mu)$is a fuzzy e-open set for any fuzzy open set $\mu \leq Y$.
(4) $F^-(\mu)$ is a fuzzy e-closed set for any fuzzy open set $\mu \in Y$.
(5) for each $x_\epsilon \in X$and for each net $(x_{\epsilon_e}^e)$ which e-converges to $x_\epsilon$in X and for each fuzzy open set $\mu \leq Y$ such that $x_\epsilon \in F^+(\mu)$,the net $(x_{\epsilon_\epsilon}^e)$ is eventually in $F^+(\mu)$.

Proof. (1) ⇔ (2) this statement is obvious.
(1) ⇔(3) Let $x_\epsilon \in F^+(\mu)$, and let μ be a fuzzy open set. It follows from (1) that there exists a fuzzy e-open set ρ_{x_ϵ} containing $x_\epsilon$such that $\rho_{x_\epsilon} \leq F^+(\mu)$.It follows that $F^+(\mu) = \vee_{x_\epsilon \in F^+(\mu)} \rho_{x_\epsilon}$ and hence $F^+(\mu)$ is fuzzy e-open. The converse can be shown easily.
(3) ⇒(4) Let $\mu \leq Y$ be a fuzzy open set. We have $Y \setminus \mu$ is a fuzzy openset. From (3), $F^+(Y \setminus \mu) = X \setminus F^-(\mu)$ is a fuzzy e-open set. Then it is obtained that $F^-(\mu)$is a fuzzy e-closed set.
(1) ⇒(5) Let $(x_{\epsilon_e}^e)$ be a net which e converges to x_ϵ in X and let $\mu \leq Y$ be any fuzzy open set such that $x_\epsilon \in F^+(\mu)$. Since F is an upper fuzzy e-continuous multifunction, it follows that there exists a fuzzy e-open set $\rho \leq X$containing x_ϵ such that $\rho \leq F^+(\mu)$. Since $(x_{\epsilon_e}^e)$e-converges to $x_{\epsilon,}$ it follows that there exists an index $e_0 \in J$ such that $(x_{\epsilon_e}^e) \in \rho$ for all $e \geq e_0$ from here,we obtain that $(x_{\epsilon_e}^e) \in \rho \leq F^+(\mu)$ for all $e \geq e_0$. Thus the net $(x_{\epsilon_e}^e)$is eventually in $F^+(\mu)$.
(5) ⇒(1) Suppose that is not true. There exists a point x_ϵ and a fuzzy openset μ with $x_\epsilon \in F^+(\mu)$ such that $\rho \nleq F^+(\mu)$ for each fuzzy e-open set $\rho \leq X$containingx_ϵ.Let $x_{\epsilon_\rho} \in \rho$ and $x_\epsilon \notin F^+(\mu)$ for each fuzzy e-open set $\rho \leq X$containing x_ϵ. Then for the e-neighborhood net $(x_{\epsilon_e}),x_{\epsilon_e} \to_e x_\epsilon$,but (x_{ϵ_e}) is not eventually in $F^+(\mu)$. This is a contradiction. Thus, F is an upper fuzzy$e$-continuous multifunction.

Theorem 3.2. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topologicalspace (X, τ) to a fuzzy topological space (Y, ν). Then the following statements are equivalent:

(1) F is lower fuzzy e-continuous.
(2) For each $x_\epsilon \in X$ and for each fuzzy open set μ such that $x_\epsilon \in F^-(\mu)$there exists a fuzzy e-open set ρ-containing x_ϵ such that $\rho \leq F^-(\mu)$.
(3) $F^-(\mu)$is a fuzzy e-open set for any fuzzy open set $\mu \leq Y$.
(4) $F^+(\mu)$is a fuzzy e-closed set for any fuzzy open set $\mu \leq Y$.
(5) For each $x_\epsilon \in X$ and for each net $(x_{\epsilon_e}^e)$ which e-converges to $x_\epsilon$in $X$and for each fuzzy open set μ ≤ Y such that $x_\epsilon \in F^-(\mu)$, the net $(x_{\epsilon_e}^e)$ is eventually in $F^-(\mu)$.

Proof. It can be obtained similarly as Theorem 3.1.

Theorem 3.3. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological(X, τ) to a fuzzy topological space (Y, ν) and let $F(X)$ be endowed with subspace fuzzy topology. If F is an upper fuzzy e-continuous multifunction, then $F : X \to F(X)$ is an upper fuzzy e-continuous multifunction.

Proof. Since $F$is an upper fuzzy e-continuous,
$F(X \wedge F(X)) = F^{+}(\mu) \wedge F^{+}(F(X)) = F^{+}(\mu)$
is fuzzy e-open for each fuzzy open subset μ of$Y$. Hence$F : X \to F(X)$ is an upper fuzzy e-continuous multifunction.

Definition 3.3. Suppose that (X,τ), (Y,ν) and (Z,ω) are fuzzy topological spaces. It is known that if F_1 : X → Y and $F_2 : Y \to Z$ are fuzzy multifunctions, then the fuzzy multifunction $F_1 \circ F_2 : X \to Z$ is defined by $(F_1 \circ F_2)(x_\epsilon) = F_2(F_1(x_\epsilon))$ for each $x_\epsilon \in X$.

Theorem 3.4. Let (X,τ), (Y,ν) and (Z,ω) be fuzzy topological spaces and let $F : X \to Y$ and $G : Y \to Z$ be fuzzy multifunction. If $F : X \to Y$ is an upper (lower) fuzzy continuous multifunction and $G : Y \to Z$ is an upper (lower) fuzzy e-continuous multifunction.Then $G \circ F : X \to Z$ is an upper (lower) fuzzy e-continuous multifunction.

Proof. Let $\lambda \leq Z$ be any fuzzy open set. From the definition of $G \circ F$, we have $(G \circ F)^{+}(\lambda) = F^{+}(G^{+}(\lambda))$
$(G \circ F)^{-}(\lambda) = F^{-}(G^{-}(\lambda)))$, since G is an upper (lower) fuzzy e-continuous, it follows that $G^{+}(\lambda) G^{-}(\lambda)$ is a fuzzy open set. Since F is an upper (lower) fuzzy continuous, it follows that $F^{+}(G^{+}(\lambda))\,(F^{-}(G^{-}(\lambda))\,)$ is a fuzzy e-open set, this shows that $G \circ F$ is an upper (lower) fuzzy e-continuous.

Theorem 3.5. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X,τ) to a fuzzy topological space (Y,ν).If F is a lower (upper) fuzzy e-continuous multifunction and $\mu \leq X$ is a fuzzy set, then the restriction multifunction $F|\mu : \mu \to Y$ is an lower (upper) fuzzy e-continuous multifunction.

Proof. Suppose that $\beta \leq Y$ is a fuzzy open set. Let $x_\epsilon \in \mu$ and let $x_\epsilon \in F^{-}|\mu(\beta)$. Since F is a lower fuzzy e-continuous multifunction, it follows that there exists a fuzzy open set $x_\epsilon \in \rho$ such that $\rho \leq F^{-}(\beta)$. From here we obtain that $x_\epsilon \in \rho \wedge \mu$ and $\rho \wedge \mu \leq F|\mu(\beta)$. Thus, we show that the restriction multifunction $F|\mu$ is lower fuzzy e-continuous multifunction. The proof for the case of the upper fuzzy e-continuity of the multifunction $F|\mu$ is similar to above.

Theorem 3.6. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological (X,τ) to a fuzzy topological space (Y,ν), let $\{\lambda_\gamma : \gamma \in \Phi\}$be a fuzzy open cover of X. If the restriction multifunction $F_\gamma = F_{\lambda_\gamma}$ is lower (upper) fuzzy e-continuous multifunction for each $\gamma \in \Phi$,then F is lower (upper) fuzzy e-continuous multifunction.

Proof. Let $\mu \leq Y$ be any fuzzy open set. SinceF_γ is lower fuzzy e-continuous for each γ, we know that $F_\gamma^{-}(\mu) = int_{\lambda_\gamma}(F_\gamma^{-}(\mu))$ and from here $F^{-}(\mu) \wedge \lambda_\gamma \leq int_{\lambda_\gamma}(F_\gamma^{-}(\mu)) \wedge \lambda_\gamma)$ and $F^{-}(\mu) \wedge \lambda_\gamma \leq int(F_\gamma^{-}(\mu)) \wedge \lambda_\gamma)$. Since $\{\lambda_\gamma : \lambda \in \Phi\}$is a fuzzy open cover of X.It follows that $F^{-}(\mu) \leq Int(F^{-}(\mu))$. Thus we obtain that F is lower (upper) fuzzy e-continuous multifunction. The proof of the upper fuzzy e-continuity of F is similar to the above.

Definition 3.4. Suppose that $F : X \to Y$ is a fuzzy multifunction from a fuzzy topological space X to a fuzzy topological space Y. The fuzzy graph multifunction $G_F : X \to X \times Y$ of F is defined as $G_F(x_\epsilon) = \{x_\epsilon\} \times F(x_\epsilon)$.

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Theorem 3.7. Let $F : X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X,τ) to a fuzzy topological space (Y,ν). If the graph function of F is lower (upper) fuzzy e-continuous multifunction, then F is lower (upper) fuzzy e-continuous multifunction.

Proof. For the fuzzy sets $\beta \le X$, $\eta \le Y$, we take

$$(\beta \times \eta)(z,y) = \begin{cases} 0 & if z \notin \beta \\ \eta(y) & if z \in \beta \end{cases}$$

Let $x_\epsilon \in X$ and let $\mu \in Y$ be a fuzzy open set such that $x_\epsilon \in F^-(\mu)$. We obtain that $x_\epsilon \in G_F^-(X \times \mu))$ and $X \times \mu$ is a fuzzy open set. Since fuzzy graph multifunction $G_F$is lower fuzzy e-continuous, it follows that there exists a fuzzy e-open set $\rho \le X$ containing x_ϵ such that $\rho \le G_F^-(X \times \mu))$. From here, we obtain that $\rho \le F^-(\mu)$. Thus, F is lower fuzzy e-continuous multifunction. The proof of the upper fuzzy e-continuity of F is similar to the above.

Theorem 3.8. Suppose that (X,τ) and $(X_{e,}\tau_e)$ are fuzzy topological space where $e \in J$. Let $F : X \to \prod_{e\in J} X_e$ be a fuzzy multifunction form X to the product space$\prod_{e\in J} Xe$ and let $P_e: \prod_{e\in J} X_e \to X_e$ be the projection multifunction for each $e \in J$ which is defined be $P_e(x_e) = \{x_e\}$. If F is an upper (lower) fuzzy e-continuous multifunction, then $P_e \circ F$ is an upper (lower) fuzzy e-continuous multifunction for each $e \in J$.

Proof. Take any $e_0 \in J$. Let μ_{e_0} be a fuzzy open set in $(X_{e,}\tau_\gamma)$. Then $(P_{e_0} \circ F)^+(\mu_{e_0}) = F^+\left(P_{e_0}^+(\mu_{e0})\right) = F^+(\mu_{e_0} \times \prod_{e\ne e_0} X_e)$

(resp. $(P_{e_0} \circ F)^-(\mu_{e_0}) = F^-\left(P_{e_0}^-(\mu_{e0})\right) = F^-(\mu_{e_0} \times \prod_{e\ne e_0} X_e)$

. Since F is upper (lower) fuzzy e-continuous multifunction and since$\mu_{e_0} \times \prod_{e\ne e_0} X_e$ is a fuzzy open set, it follows that $F^+(\mu_{e_0} \times \prod_{e\ne e_0} X_e)$ (resp. $F^-(\mu_{e_0} \times \prod_{e\ne e_0} X_e)$ is fuzzy e-open in (X,τ). It shows that $P_{e_0} \circ F$ is upper (lower) fuzzy e-continuous multifunction. Hence we obtain that $P_e \circ F$ is an upper (lower) fuzzy e-continuous multifunction for each $e \in J$.

Theorem 3.9. Suppose that for each $e \in J$, $(X_{e,}\tau_e)$ and $(Y_{e,}\eta_e)$ are fuzzy topological spaces. Let $P_e: X_e \to Y_e$be a fuzzy multifunction for each $e \in J$ and let $F: \prod_{e\in J} X_e \to \prod_{e\in J} Y_e$ be defined by $((x_e)) = \prod_{e\in J} F_e(x_e)$ from the product space $\prod_{e\in J} X_e$to product space $\prod_{e\in J} Y_e$. If F is an upper (lower) fuzzy e-continuous multifunction, then each F_e is an upper (lower) fuzzy e-continuous multifunction for each $e \in J$.

Proof. Let $\mu_e \le Y_e$be a fuzzy open set. Then $\mu_e \times \prod_{e\ne\beta} Y_\beta$ is a fuzzy open set. Since F is an upper (lower) fuzzy e-continuous multifunction, it follows that $F^+\left(\mu_e \times \prod_{e\ne\beta} Y_\beta\right) = F^+(\mu_e) \times \prod_{e\ne\beta} Y_{\beta,}$ $(F^-\left(\mu_e \times \prod_{e\ne\beta} Y_\beta\right) = F^-(\mu_e) \times \prod_{e\ne\beta} X_\beta$
) is a fuzzy e-open set. Consequently, we obtain that $F^+(\mu_e)(F^-(\mu_e))$is a fuzzy e-open set. Thus, we show that F_e is an upper (lower) fuzzy e-continuous multifunction.

Theorem 3.10. Suppose that $(X_1,\ \tau_1),(X_{2,}\tau_2),(Y_1,\ \nu_1)$ and $(Y_2,\ \nu_2)$are fuzzy topological spaces and $F_1: X_1 \to Y_1$, $F_2 : X_2 \to Y_2$are fuzzy multifunctions and suppose that if $\eta \times \beta$ is fuzzy e-open set then η and β are fuzzy e-open sets for any fuzzy sets $\eta \le Y_{1,}\ \beta \le Y_2$. Let $F_1 \times F_2: X_1 \times X_2 \to Y_1 \times Y_2$be a fuzzy multifunction which is defined by $(F_1 \times F_2)(x_\epsilon, y_\nu) = F_1(x_\epsilon) \times F_2(y_\nu)$. If

$F_1 \times F_2$ is an upper (lower) fuzzy e-continuous multifunction, then $F_1$and $F_2$are upper (lower) fuzzy e-continuous mutifunctions.

Proof. We know that $(\mu^* \times \beta^*)(x_\epsilon, y_v) = \min\{\mu^*(x), \beta^*(y)\}$ for any fuzzy sets μ^*, β^* and for any fuzzy point x_ϵ, y_v. Let $\mu \times \beta \leq Y_1 \times Y_2$ be a fuzzy open set. It known that $(F_1 \times F_2)^+(\mu \times \beta) = F_1^+(\mu) \times F_2^+(\beta)$. Since $F_1 \times F_2$ is an upper fuzzy e-continuous multifunction, it follows that $F_1^+(\mu) \times F_2^+(\beta)$is afuzzy e-open set. From here, $F_1^+(\mu)$ and $F_2^+(\beta)$are fuzzy e-open sets. Hence, it is obtain that F_1 and F_2 are upper fuzzy e-continuous multifunctions. The proof of the lower fuzzy e-continuity of the multifunction F_1 and F_2 is similar to the above.

Theorem 3.11. Suppose that (X, τ), (Y, v) and (Z, ω) are fuzzy topological spaces and $F_1: X \to Y$, $F_2: Y \to Z$ are fuzzy multifunction and suppose that if $\eta \times \beta$ is a fuzzy e-opens set, then η and β are fuzzy e-open sets for any fuzzy set $\eta \leq Y, \beta \leq Z$. Let $F_1 \times F_2: X \to Y \times Z$ be a fuzzy multifunction which is defined by $(F_1 \times F_2)(x_\epsilon) = F_1(x_\epsilon) \times F_2(x_\epsilon)$. If $F_1 \times F_2$ is an upper (lower) fuzzy e-continuous multifunction, then F_1 and $F_2$are upper (lower) fuzzy e-continuous multifunction.

Proof. Let $x_\epsilon \in X$ and let $\mu \leq \beta$, $\beta \leq Z$ be fuzzy e-open sets such that $x_\epsilon \in F_1^+(\mu)$ and $x_\epsilon \in F_2^+(\beta)$. Then we obtain that $F_1(x_\epsilon) \leq \mu$ and $F_2(x_\epsilon) \leq \beta$ and from here, $F_1(x_\epsilon) \times F_2(x_\epsilon) = (F_1 \times F_2)(x_\epsilon) \leq \mu \times \beta$. We have $x_\epsilon \in (F_1 \times F_2)^+(\mu \times \beta)$. Since $F_1 \times F_2$ is an upper fuzzy e-continuous multifunction, it follows that there exist a fuzzy e-open set ρ containing x_ϵ such that $\rho \leq (F_1 \times F_2)^+(\mu \times \beta)$. We obtain that $\rho \leq F_1^+(\mu)$and $\rho \leq F_2^+(\beta)$. Thus we obtain that F_1 and $F_2$are fuzzy e-continuous multifunctions. The proof of the lower fuzzy e-continuity of the multifunctionF_1 andF_2 is similar to the above.

References

1. K. K. Azad, “On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity”, J. Math. Anal. Appl., **1981,** 82, 297–305.
2. C. L. Chang, “Fuzzy topological space”, J. Math. Anal. Appl., **1968**, 24, 182–190.
3. P. Pao-Ming and L. Ying-Ming, “Fuzzy topology I. neighborhood structure of a fuzzy point and Moore-simith convergence”, J. Math. Anal. Appl., **1980**, 76, 571–599.
4. A. Mukherjee and S.Debnath, “δ-semi-open sets in fuzzy setting”, J. Tri. Math.Soc.,**2006**, 8, 51–54.
5. V. Seenivasan and K. Kamala, “Fuzzy e-continuity and fuzzy e-open sets”, Annals of Fuzzy Mathematics and Informatics, **2014**, 8 (1), 141–148.
6. L. A. Zadeh, “Fuzzy sets”, Information and Control, **1965**, 8, 338–353.